

Mercury's spin-orbit model and signature of its dynamical parameters

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Abstract. The 3:2 spin-orbit resonance between the rotational and orbital motions of Mercury results from a functional dependance on a tidal friction adding to a non-zero eccentricity with a permanent asymmetry in the equatorial plane of the planet. The upcoming space missions, MESSENGER and BepiColombo with onboard instrumentation capable of measuring the Mercury's rotational parameters, stimulate the objective to attempt to an accurate theory of the planet's rotation. We have used our BJV relativistic model of solar system integration including the spin-orbit motion of the Moon. This model had been previously built in accordance with the requirements of the Lunar Laser Ranging observational accuracy. We extended this model to the spin-orbit couplings of the terrestrial planets including Mercury; the updated model is called SONYR (acronym of Spin-Orbit N-BodyY Relativistic model). An accurate rotation of Mercury has been then obtained. Moreover, the conception of the SONYR model is suitable for analyzing the different families of hermean rotational librations. We accurately identify the non-linear relations between the rotation of Mercury and its dynamical figure (C/MR^2 , C_{20} , and C_{22}). Notably, for a variation of 1% on the C/MR^2 value, signatures in the φ hermean libration in longitude as well as in the η obliquity of the planet are respectively 0.45 arcseconds (as) and 2.4 milliarcseconds (mas). These determinations provide new constraints on the internal structure of Mercury to be discussed with the expected accuracy forecasted in the BepiColombo mission (respectively 3.2 and 3.7 as according to Milani *et al.* 2001).

Key words. libration – rotation – Mercury – principal figure

1. Introduction

Pettengill and Dyce discovered in 1965 the 3:2 spin-orbit resonance state of Mercury (the rotational and orbital periods are 56.646 and 87.969 days respectively). This particular resonance results from a functional dependance on a tidal friction adding to a non-zero eccentricity with a permanent asymmetry in the equatorial plane of the planet. Besides, in a tidally evolved system, the spin pole is expected to be trapped in a Cassini state: the orbital and rotational parameters are indeed matched in such a way that the spin pole, the orbit pole, and the solar system invariable pole remain coplanar while the spin and orbital poles on average precess at the same rate (Colombo 1965; Peale 1969, 1973).

The upcoming missions, MESSENGER (Solomon *et al.* 2001) and BepiColombo (Milani *et al.* 2001) with onboard instrumentation capable of measuring the rotational parameters of Mercury stimulate the objective to attempt to an accurate theory of the Mercury's rotation. The current method for obtaining constraints on the state and structure of the hermean nucleus is based on the as-

sertions introduced by Peale in 1976. According to this author, the determination of the four parameters, namely C_{20} , C_{22} , η and φ , should be sufficient to constrain size and state of the Mercury's core (C_{20} and C_{22} are spherical harmonics of the second degree, η is the obliquity of Mercury, and φ is the libration in longitude of 88 days).

In the first section of this paper, we present our approach which takes into account the coupled spin-orbit motion such as it derives from the integration of the entire solar system. In the second section, we present the dynamical behavior of the Mercury's spin-orbit motion. In the last section, following our previous analysis of the different families of hermean rotational librations (see our previous paper Rambaux and Bois 2003), we make in evidence some accurate signatures due to parameters related to the dynamical figure of Mercury (C/MR^2 , C_{20} , and C_{22}) in the librations as well as in the obliquity of the planet. We find 0.45 arcseconds (as) and 2.4 milliarcseconds (mas) respectively on φ and η for a variation of 1% on the C/MR^2 value. These determinations provide new constraints on the internal structure of Mercury to be discussed with the expected accuracy forecasted in the

BepiColombo mission (respectively 3.2 and 3.7 as according to Milani *et al.* 2001).

2. The Spin-Orbit N-Body Relativistic model

For obtaining the real motion of Mercury, we have used our BJV model of solar system integration including the coupled spin-orbit motion of the Moon. This model, expanded in a relativistic framework, had been previously built in accordance with the requirements of the Lunar Laser Ranging observational accuracy (Bois 2000; Bois & Vokrouhlický 1995). We extended the BJV model by generalizing the spin-orbit couplings to the terrestrial planets (Mercury, Venus, Earth, and Mars). The model is at present called SONYR (acronym of Spin-Orbit N-BodY Relativistic model). As a consequence, the SONYR model gives an accurate simultaneous integration of the spin-orbit motion of Mercury. The integration of the solar system, including the Mercury's spin-orbit motion, uses a global reference system given by the solar system barycenter. The model is solved by modular numerical integration and controlled in function of the different physical contributions and parameters taken into account. It permits the analysis of the different families of librations and the identification of their causes (such as the planetary interactions) or their interdependences with the parameters involved in the dynamical figure of the planet. A complete description of the SONYR model is given in our previous paper (Rambaux and Bois 2003). The spin-orbit motion of Mercury is characterized by two proper frequencies (namely $\Phi = 15.847$ years and $\Psi = 1066$ years) and its 3:2 resonance presents a second synchronism, which can be understood as a *spin-orbit secular resonance* ($\Pi = 278\,898$ years).

Using the SONYR model and its method of analysis (modular and controlled numerical integration, differentiation method), the librations are accurately surrounded and identified. Our previous paper Rambaux and Bois (2003) presents a detailed analysis of the whole spin-orbit mechanism of Mercury as well as its main librations.

3. The Mercury's spin-orbit motion

Using our SONYR model, the present work is devoted for studying the impact of the dynamical figure of Mercury on its spin-orbit motion. We have then analyzed the impact of the three dynamical parameters C/MR^2 , C_{20} , and C_{22} on the rotational motion of Mercury.

Figure 1 presents the dynamical behavior of the Mercury's rotation expressed by the Euler angles ψ , θ , and φ (three first panels) as well as its obliquity η plotted in the fourth panel over 500 days. The Euler angles (ψ , θ , φ) related to the 3-1-3 angular sequence describe the evolution of the body-fixed axes $Oxyz$ with respect to the axes of the local reference frame $OXYZ$. Let us recall the definition used for these angles : ψ is the precession angle of the polar axis Oz around the reference axis OZ , θ is the nutation angle representing the inclination of Oz

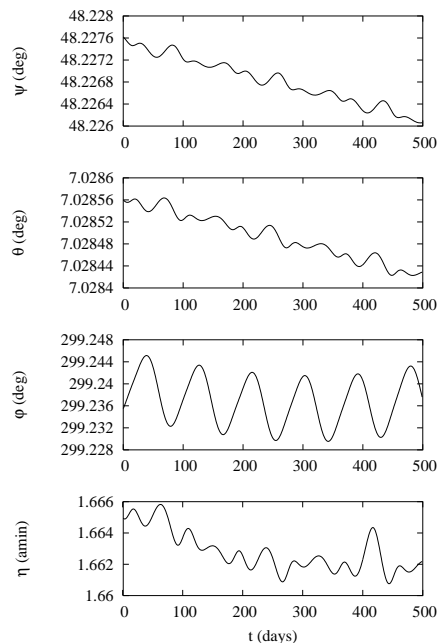


Fig. 1. The rotational motion of Mercury expressed in the classical 3-1-3 Eulerian sequence (ψ , θ , φ) with respect to the ecliptic reference frame. In the bottom panel is plotted the obliquity η . The four curves are plotted over 500 days (horizontal axes). Degrees are on the vertical axes for ψ , θ , and φ , while the vertical axis for η is plotted in arcminutes (amin).

with respect to OZ , and φ is the rotation around Oz and conventionally understood as the rotation of the greatest energy (it is generally called the proper rotation). The axis of inertia around which is applied the proper rotation is called the axis of figure and defines the North pole of the rotation (Bois 1992). Let us remark that in this Figure, we have removed the mean rotation of 58.646 days in the φ angle in order to better distinguish the librations. The resulting amplitude of libration appearing in φ is 20 as, which is in good agreement with Balogh and Giamperi (2002).

The integration of the SONYR model including the simultaneous spin-orbit motion of Mercury gives also the dynamical behavior of the hermean obliquity by the way of the following relation :

$$\cos \eta = \cos i \cos \theta + \sin i \sin \theta \cos(\Omega - \psi) \quad (1)$$

where i and Ω are respectively the inclination and ascending node of the Mercury's orbital plane relative to the ecliptic plane (the obliquity is the angle between the polar axis and the orbital plane). In our computations presented in this paper, the used initial conditions are listed in Table 1. For η , $\dot{\psi}$, and $\dot{\theta}$, we use the values determined in our previous work (Rambaux and Bois 2003).

The dynamical structure of the Mercury's rotation given in Figure 1 results from the direct solar torque whose coordinates include the planetary interactions. The effect

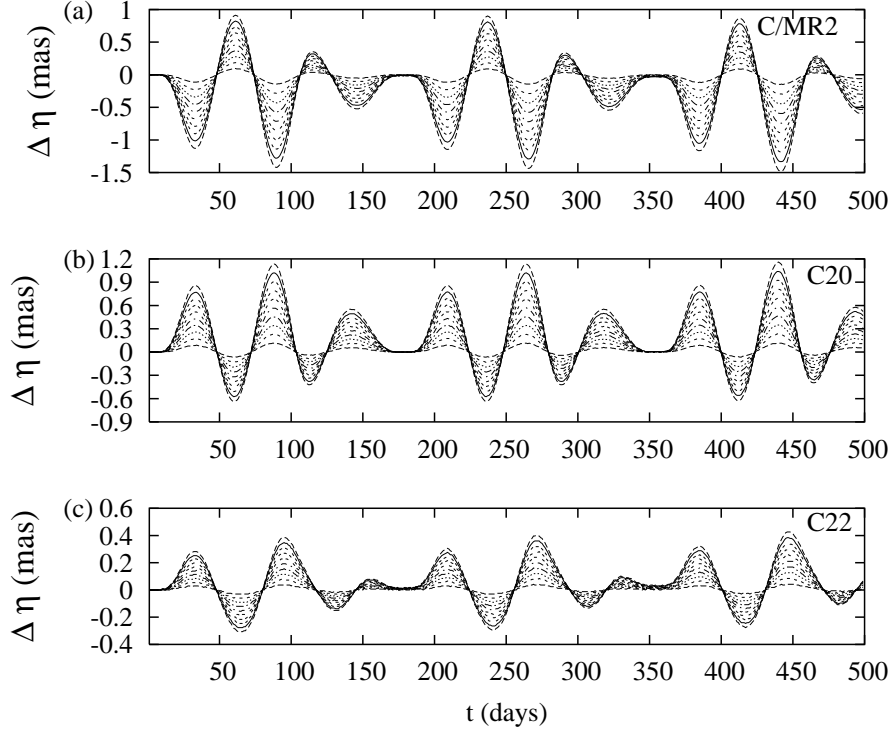


Fig. 2. Signatures of the variations (with a step equal to 0.1%) of the dynamical parameter values on the Mercury's obliquity: (a) C/MR^2 , (b) C_{20} , and (c) C_{22} . On each panel, 10 curves are plotted over 500 days; the maximum of amplitude is obtained for 1%. Milliarcseconds (mas) are on the vertical axes.

of the gravitational torque due to Venus is five orders of magnitude smaller than the one due to the Sun. The P_φ rotation period of 58.646 days appears in the ψ and θ angles, as well as in η . Signature of the P_λ orbital period of 87.969 days is clearly visible in the φ angle (this angle is called in literature the libration in longitude of 88 days). A third period appears in the ψ , θ , and η angles, namely 176.1 days. This period \tilde{P} results from the 3:2 spin-orbit resonance ($\tilde{P} = 2P_\lambda = 3P_\varphi$).

Table 1. Our initial conditions at 07.01.1969 (equinox J2000): (a) Mean values derived from the SONYR model; (b) Seidelmann *et al.* (2002). In addition, our SONYR value of the Mercury's obliquity is 1.6 amin.

Mercury			
Rotation angles			
ψ	=	48.386 deg	$\dot{\psi}$ = -0.616 10^{-7} deg/day (a)
θ	=	7.031 deg	$\dot{\theta}$ = -0.267 10^{-8} deg/day (a)
φ	=	299.070 deg	$\dot{\varphi}$ = 6.138505 deg/day (b)

4. The impact of the dynamical parameters

The gravity field of Mercury is globally unknown. The tracking data from the three fly-by of Mariner 10 in 1974-75 have been re-analyzed by Anderson *et al.* (1987) in

order to give an accurate estimation of the normalized coefficients C_{20} and C_{22} . The resulting nominal values are $C/MR^2 = 0.34$, $C_{20} = (6.0 \pm 2.0) 10^{-5}$, and $C_{22} = (1.0 \pm 0.5) 10^{-5}$. As a consequence, the dynamical coefficients $\alpha = (C - B)/A$, $\beta = (C - A)/B$, and $\gamma = (B - A)/C$ (where A, B , and C are the three principal moments of inertia of Mercury) are inferred from C_{20} and C_{22} by the following formulae :

$$\begin{aligned}
 \alpha &= \frac{-C_{20} - 2C_{22}}{C_{20} - 2C_{22} + C/MR^2} \\
 \beta &= \frac{-C_{20} + 2C_{22}}{C_{20} + 2C_{22} + C/MR^2} \\
 \gamma &= \frac{4C_{22}}{C/MR^2}
 \end{aligned} \tag{2}$$

In these conditions, we may assume the uncertainties on the Mercury's dynamical parameter values as dynamical variations on the Mercury's rotation. We have computed the impact of the variations of the greatest principal moment of inertia, C/MR^2 , as well as C_{20} and C_{22} , both on the obliquity and the libration in longitude. Figures 2 and 3 present the results plotted over 500 days.

These Figures show three panels corresponding to signatures of the variations of C/MR^2 (a), C_{20} (b), and C_{22} (c) on η and φ . In each panel, 10 integrations with a step of 0.1% over the given parameter are plotted over 500 days; the maximum of amplitude is obtained for 1%. In each

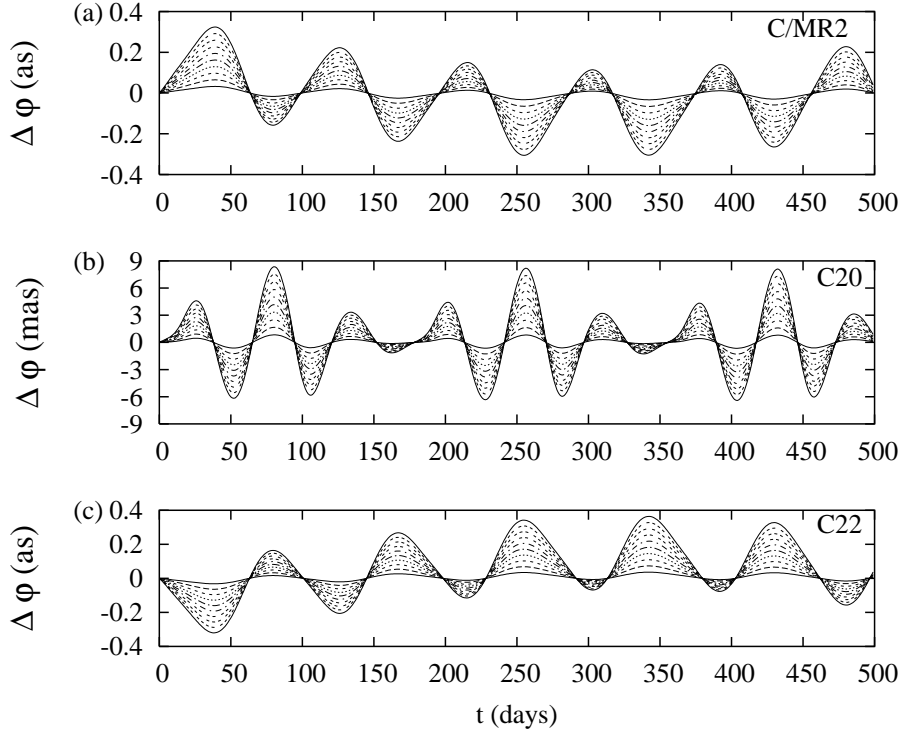


Fig. 3. Signatures of the variations (with a step equal to 0.1%) of the dynamical parameter values on the Mercury's libration in longitude: (a) C/MR^2 , (b) C_{20} , and (c) C_{22} . On each panel, 10 curves are plotted over 500 days; the maximum of amplitude is obtained for 1%. Arcseconds (as) are on the vertical axes of the top and bottom panels while it is about milliarcseconds (mas) in the middle panel.

plot, the maximum amplitude is reached with the characteristic period of 176.1 days for the obliquity η , and 87.969 days for the libration in longitude φ . The results related to variations of 1% are listed in Table 2.

Table 2. Results of the 1% variations of the dynamical parameters on the Mercury's obliquity and libration in longitude.

	C/MR^2	C_{20}	C_{22}
η	2.4 mas	1.8 mas	0.7 mas
φ	0.45 as	14 mas	0.45 as

In the Figure 2, a variation of 1 % on the C/MR^2 parameter induces a peak to peak amplitude of 2.4 mas in the obliquity, whereas with the C_{20} and C_{22} coefficients, the amplitudes are 1.8 and 0.7 mas respectively. The characteristic period of these signatures is $\tilde{P} = 176.1$ days, which expresses the spin-orbit coupling of Mercury. The dynamical behavior of η proves to be the right tracer for analyzing the relationship between the rotation of Mercury and its dynamical figure.

In the Figure 3, the impact of variations of the C_{20} value on the libration in longitude is 30 times smaller than the one due to variations of C/MR^2 and C_{22} . This is make

clear by the fact that the dynamical parameter γ depends only on C_{22} and C/MR^2 . As a consequence, C_{20} has only an indirect action on the φ angle. In addition, the dynamical behaviors in Figure 3, panels (a) and (c), are symmetrical; this fact reflects structure of the third equation in the formulae (2). The relation between C_{20} and φ proves to be a good tracer of the \tilde{P} spin-orbit period while the relations between C/MR^2 and φ , or C_{22} and φ , express the P_λ period.

5. Conclusion

One important objective of the upcoming space missions, MESSENGER and BepiColombo, is to constrain the nature of the Mercury's core. The theoretical framework follows the procedure proposed by Peale (1976) and Peale *et al.* (2002). The onboard instrumentations are planned for measuring the Mercury's rotational parameters with a great accuracy. However, our work shows that for obtaining an excepted accuracy up to 1% on the C/MR^2 coefficient, the measurements have to reach at least 0.45 as on the Mercury's rotation and 2.4 mas on the obliquity. To our knowledge, these high accuracies are not yet achieved within the simulations.

The Mercury's rotation strongly depends on the C/MR^2 (0.45 as) and C_{22} (0.45 as) coefficients. Besides, the obliquity of Mercury hierarchically depends on C/MR^2

(2.4 mas), C_{20} (1.8 mas), and C_{22} (0.7 mas). The dynamical behavior of Mercury's obliquity is finally the tracer for analyzing the non-linear relations between the rotation of Mercury and parameters of its dynamical figure.

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